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Elastic constants and surface acoustic waves of superlattices

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Abstract. Elastic constants of superlattices and their surface acoustic waves are theoretically studied. An exact expression to calculate the surface acoustic wave velocity dispersions is derived for superlattices of finite thickness consisting of two kinds of layers rigidly stacked upon each other. The formulations are performed for both free superlattice plates and substrated superlattices. Numerical calculations for Cu/Al and Cu/Ag systems are compared with those obtained using the effective elastic constant (EEC) model (Grimsditch M 1985 *Phys. Rev.* B **31** 6818). A comparison clarifies the applicability of the EEC model.

1. Introduction

Since the first report of the 'supermodulus effect' [1], an anomalous increase of more than 200% of various elastic properties of metallic superlattices and multilayers, the elastic properties of metallic superlattices have attracted much attentions [2–5]. Metallic superlattices are usually prepared by means of sputtering or evaporation [5]. Such superlattices have a strong tendency to form a 'pencil-type texture', i.e. one in which the grains have a common orientation normal to the film but are randomly oriented within the film plane [6].

Brillouin scattering from surface acoustic waves has been successfully applied to the study of the elastic properties of metallic superlattices and multilayers for the past 20 years [7]. In order to understand their inelastic properties one needs to know the elastic constants of such superlattices. The effective elastic constants for a periodically laminated structure of orthorhombic symmetry have been derived by Grimsditch [7, 8]. The effective elastic constant (EEC) model is valid in the long-wavelength regime, i.e. it can be applicable for superlattices with much shorter period compared with phonon wavelength. Here 'period' means the shortest unit of layers which consist of the constituents of periodically layered structures. However, the definition of 'short period' does not seem to be clear, although the EEC model is often used in the literature [5, 9]. Experiments usually cover a wide range of period lengths (10–1000 Å). Therefore, the development of a general formula of surface acoustic waves in a periodically layered structure beyond the EEC model is required.

Many theoretical studies of the elastic waves of a superlattice have been carried out in the last 15 years and this subject is considered to be well established [10–14]. However, in the conventional theory [11–14] superlattices are supposed to be infinite or semi-infinite along the stacking direction, for which one can simplify the theoretical approach by assuming translational symmetry. In contrast, we consider superlattice plates and substrated superlattices of finite thickness and discuss the surface elastic waves known as the Rayleigh wave and the Sezawa waves [10, 15]. Mathematically, the dispersion equation for the surface waves is equivalent to the condition that the determinant of a 4×4 boundary condition matrix is zero. Although our formulation gives both the Rayleigh wave and the Sezawa waves, we restrict our discussion to the Rayleigh wave in this paper. It is possible to treat a superlattice as a medium with EECs (the EEC model) [5, 8]. In this approach, the surface waves can be obtained through much easier calculations than the direct and complicated calculations performed in the present paper.

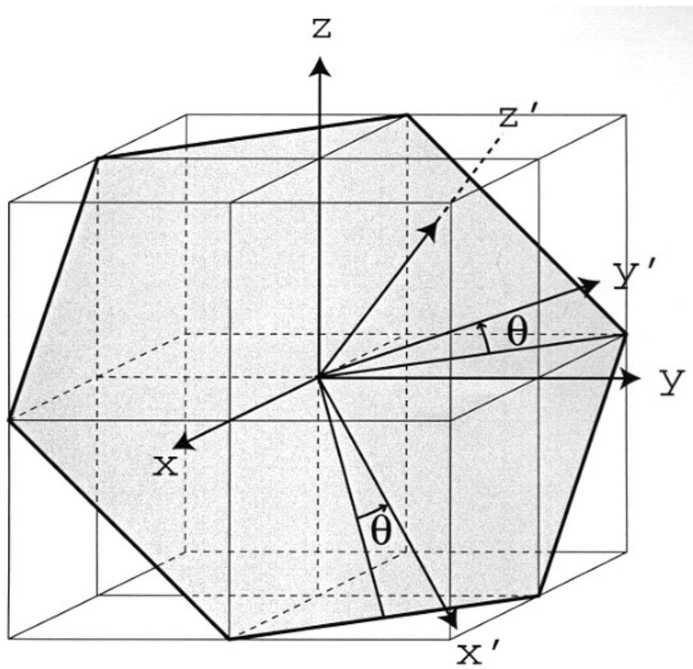


Figure 1. The transformation from the cubic crystallographic coordinates (x, y, z) into the (111) film coordinates (x', y', z'). The shaded hexagonal plane indicates the (111) film plane.

Our purpose is to derive the exact dispersion equation for the surface waves in a superlattice, and consequently to clarify the applicability of the EEC model. We will consider only cubic materials as constituents of a superlattice and the (111) planes as their basal plane. Some numerical calculations are performed for Cu/Al and Cu/Ag superlattices using both approaches.

2. Film geometric elastic constants

We introduce two kinds of coordinate systems: the crystallographic coordinate ($x_1 = x$, $x_2 = y$, $x_3 = z$) and the film geometric coordinate ($x'_1 = x'$, $x'_2 = y'$, $x'_3 = z'$) [6]. The axis z' is always normal to the film plane, and the x' and y' axes are orthogonal but in arbitrary orientations within the film plane. The transformation from the (x, y, z) coordinate system to the (x', y', z') coordinate system can be symbolically written using the rotational matrix $\mathbf{A}$ as $x'_i = A_{ij}x_j$. Here and hereafter the usual convention regarding summation on repeated subscripts is used [15, 16]. The primary elastic modulus tensor C is related to the film geometric elastic modulus tensor C' by the following tensor transformation equation [15, 16]:

$$C'_{pqrs} = A_{pi}A_{qj}A_{rk}A_{sl}C_{ijkl}. \tag{1}$$

We have only three non-zero components of C in the cubic system [16]: $C_{iiii} \equiv C_{11}$, $C_{iijj} \equiv C_{12}$ and $C_{ijij} \equiv C_{44}$ $(i, j = x, y, z, i \neq j)$. Using expression (1), we can obtain the elastic constants for an arbitrary film coordinate system. We discuss the case of the (111) film plane in the following sections.

The coordinates in the (111) film plane can be derived by transforming the crystallographic unit vectors $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ into new unit vectors $(1/\sqrt{6}, 1/\sqrt{6}, -2/\sqrt{6})$, $(-1/\sqrt{2}, 1/\sqrt{2}, 0)$ and $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, and then rotating around the new z' axis by an angle θ. The relation between the crystallographic coordinate system (x, y, z) and the (111) film system (x', y', z') is illustrated in figure 1. The C' elastic tensor components in this case are given in table 1. Here we notice the relation $C'_{11} - C'_{12} = 2C'_{66}$, which ensures the elastic isotropy within the (111) plane. We are considering evaporated or sputtered films in which numerous grains have a common orientation normal to the film but are randomly oriented in the film plane [6]. The θ-dependent parts of the elastic tensor components are smeared in such a situation. The angular-independent parts completely agree with those given in [5, 6].

Table 1. The elastic constant tensor C'_{ij} in the (111) basal plane ($\varepsilon = C_{11} - C_{12} - 2C_{44}$).

	j					
i	$1(x'x')$	$2(y'y')$	$3(z'z')$	$4(y'z')$	$5(z'x')$	$6(x'y')$
$1(x'x')$	$C_{11} - \frac{1}{2}\varepsilon$	$C_{12} + \frac{1}{6}\varepsilon$	$C_{12} + \frac{1}{3}\varepsilon$	$\frac{\sin 3\theta}{3\sqrt{2}}\varepsilon$	$-\frac{\cos 3\theta}{3\sqrt{2}}\varepsilon$	0
$2(y'y')$		$c_{11} - \frac{1}{2}\varepsilon$	$C_{12} + \frac{1}{3}\varepsilon$	$-\frac{\sin 3\theta}{3\sqrt{2}}\varepsilon$	$\frac{\cos 3\theta}{3\sqrt{2}}\varepsilon$	0
$3(z'z')$			$C_{11} - \frac{2}{3}\varepsilon$	0	0	0
$4(y'z')$				$C_{44} + \frac{1}{3}\varepsilon$	0	$\frac{\cos 3\theta}{3\sqrt{2}}\varepsilon$
$5(z'x')$					$C_{44} + \frac{1}{3}\varepsilon$	$\frac{\sin 3\theta}{3\sqrt{2}}\varepsilon$
$6(x'y')$						$C_{44} + \frac{1}{6}\varepsilon$

3. Elastic waves in a film

In the previous section, we discussed the elastic tensors of cubic materials in a film coordinate. Using the results we can deal with elastic waves in a bulk medium in an arbitrary coordinate system. Here we treat a film, which is a constituent of a superlattice or a substrate. At the boundary of each film a propagating elastic wave undergoes reflection and refraction and it splits into reflected and refracted waves, except at the top and bottom surfaces, where the refracted waves do not exist, but the incident and reflected waves do. These are quasi-transverse and quasi-longitudinal waves, which form the surface acoustic waves known as the Rayleigh wave and the Sezawa waves. Prior to discussing the surface acoustic waves, we formulate the elastic wave equations within a film.

The equation of motion in an elastic medium can be written after the description of Farnell [15] in the form

$$\rho \frac{\partial^2 u_p}{\partial t^2} = C'_{pqrs} \frac{\partial^2 u_r}{\partial x_q \partial x_s} \qquad (p, q, r, s = 1, 2, 3) \tag{2}$$

where ρ is the density of the medium, u_r is the rth component of the displacement of the medium, C'_{pqrs} is an element of the elastic constant tensor in the film coordinate, and x_s corresponds to the sth Cartesian component in the film coordinate system. In the preceding section the symbol $'$ is affixed to variables and tensors in the film coordinate system $(x'_1, x'_2, x'_3) \equiv (x', y', z')$. Hereafter, we only treat the quantities in the film coordinate, so we will drop this symbol.

Since the medium is elastically isotropic in the (x, y) plane, it is enough to restrict the elastic waves propagating in the (x, z) plane. For the wave with the wavevector $\boldsymbol{q} = (q_x, 0, q_z)$ and frequency ω, its displacement at the point (x, y, z) and time t is expressed as

$$u_\alpha = U_\alpha \exp\{\mathrm{i}(q_x x + q_z z - \omega t)\} = \tilde{U}_\alpha \exp(\mathrm{i} q_z z) \qquad (\alpha = x, y, z). \tag{3}$$

We can discuss the wave equation (2) within each layer of the superlattice by use of the displacement (3). The elastic waves in a superlattice are obtained by solving these wave equations with the boundary conditions at each interface.

There have been many theoretical studies of acoustic waves of superlattices [10–14]. It is well known that the displacements describe a transverse elastic wave and sagittal elastic waves [11, 12, 15]. In fact, substituting the displacement (3) into equation (2) yields

$$\rho\omega^2 \begin{pmatrix} U_x \\ U_z \end{pmatrix} = \begin{pmatrix} \bar{C}_{11} q_x^2 + \bar{C}_{44} q_z^2 & (\bar{C}_{13} + \bar{C}_{44}) q_x q_z \\ (\bar{C}_{13} + \bar{C}_{44}) q_x q_z & \bar{C}_{44} q_x^2 + \bar{C}_{33} q_z^2 \end{pmatrix} \begin{pmatrix} U_x \\ U_z \end{pmatrix} \tag{4}$$

and

$$\rho\omega^2 U_y = (\bar{C}_{66} q_x^2 + \bar{C}_{44} q_z^2) U_y \tag{5}$$

where $\bar{C}_{pq}$ denotes the angular-independent parts of the elastic tensor C'. Equation (4) gives the sagittal modes, while the transverse mode is obtained from equation (5). Here we will treat sagittal waves. For the sake of convenience, we introduce the following parameters:

$$Q = q_z / q_x \tag{6}$$

$$U = U_z / U_x = \tilde{U}_z / \tilde{U}_x \tag{7}$$

and

$$\xi^2 = \rho\omega^2 / (\bar{C}_{44} q_x^2). \tag{8}$$

Then the elastic wave equation for the sagittal modes (4) can be expressed as

$$U = -\frac{\bar{C}_{44} Q^2 + \bar{C}_{11} - \bar{C}_{44}\xi^2}{(\bar{C}_{13} + \bar{C}_{44}) Q} = -\frac{(\bar{C}_{13} + \bar{C}_{44}) Q}{\bar{C}_{33} Q^2 + \bar{C}_{44} - \bar{C}_{44}\xi^2}. \tag{9}$$

From equation (9), we have

$$Q^4 + \{A - (1 + B)\xi^2\} Q^2 + (1 - \xi^2)(C - B\xi^2) = 0 \tag{10}$$

where A, B, and C are defined as

$$A = \frac{\bar{C}_{11}}{\bar{C}_{44}} - \frac{\bar{C}_{13}}{\bar{C}_{33}} \left(2 + \frac{\bar{C}_{13}}{\bar{C}_{44}}\right) \qquad B = \frac{\bar{C}_{44}}{\bar{C}_{33}} \qquad C = \frac{\bar{C}_{11}}{\bar{C}_{33}}. \tag{11}$$

Equation (10), a quadratic equation of Q^2, gives two solutions Q_1^2 and Q_2^2 ($|Q_1^2| \geqslant |Q_2^2|$) for a given set of ω and q_x. For a given value of Q^2, we have a forward wave ($+Q$) and a backward wave ($-Q$). Consequently equation (10) gives four solutions Q_1, $-Q_1$, Q_2 and $-Q_2$, and equation (9) determines the amplitude ratios U_1, $-U_1$, U_2, and $-U_2$ for each Q.

The elastic waves have eigenvectors given by

$$a(1, 0, U_1)f(Q_1 z) \tag{12a}$$

$$b(1, 0, -U_1)f(-Q_1 z) \tag{12b}$$

$$c(1, 0, U_2)f(Q_2 z) \tag{12c}$$

$$d(1, 0, -U_2)f(-Q_2 z) \tag{12d}$$

with

$$f(Z) = \exp(\mathrm{i}q_x Z). \tag{13}$$

Here the coefficients a, b, c, and d are the xth components of the amplitudes ($\tilde{U}_x$) for the corresponding elastic waves. The forward waves are characterized by (U_1, Q_1) and (U_2, Q_2) and the backward waves by $(-U_1, -Q_1)$ and $(-U_2, -Q_2)$, respectively. By adjusting the coefficients a to d to satisfy the stress-free boundary conditions, one obtains the surface waves of a film.

4. Elastic waves and surface waves in superlattices

We consider a superlattice occupying a space $0 \geqslant z \geqslant -z_L$ with its top surface at $z = 0$ and a substrate occupying a space $-z_L \geqslant z \geqslant -z_L - d_s$. The superlattice consists of alternating layers of thickness d_1 of constituent 1 and thickness d_2 of constituent 2. A unit spatial period is $D = d_1 + d_2$. We distinguish each constituent and a substrate by affixing the superscript (i) to the elastic constant C_{kl}, the wavevector q_z, the density ρ, the displacements u_α and U_α; $C_{kl}^{(i)}$, $q_z^{(i)}$, $\rho^{(i)}$, $u_\alpha^{(i)}$, and $U_\alpha^{(i)}$ ($i = 1$, 2 or s; s indicates a substrate). The quantities Q, U, and ξ may be symbolically distinguished by Q_i, U_i, and ξ_i.

The lth constituent i is located at $-z_{il} \geqslant z \geqslant -z_{il} - d_i$, where $z_{1l} = (l-1)D$, $z_{2l} = z_{1l} + d_1$ and $z_{s1} = z_L$. Four elastic waves exist in this region: two forward waves characterized by (U_{i1}, Q_{i1}) and (U_{i2}, Q_{i2}) and two backward waves characterized by $(-U_{i1}, -Q_{i1})$ and $(-U_{i2}, -Q_{i2})$, where the subscript i in Q_{ij} and U_{ij} denotes the ith constituent ($i = 1$, 2 or s). The displacement vector $(u_x^{(i)}, 0, u_z^{(i)})$ in this case can be written as

$$\begin{pmatrix} u_x^{(i)} \\ u_z^{(i)} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ U_{i1} & -U_{i1} & U_{i2} & -U_{i2} \end{pmatrix} P_i(z + z_{il})|u_{i,l}^{+}\rangle$$
$$= \begin{pmatrix} 1 & 1 & 1 & 1 \\ U_{i1} & -U_{i1} & U_{i2} & -U_{i2} \end{pmatrix} P_i(z + z_{il} + d_i)|u_{i,l}^{-}\rangle \tag{14}$$

where we use the definitions

$$P_i(z) = \begin{pmatrix} f(Q_{i1}z) & 0 & 0 & 0 \\ 0 & f(-Q_{i1}z) & 0 & 0 \\ 0 & 0 & f(Q_{i2}z) & 0 \\ 0 & 0 & 0 & f(-Q_{i2}z) \end{pmatrix} \tag{15}$$

and

$$|u_{i,l}^{\pm}\rangle = \begin{pmatrix} a_{il}^{\pm} \\ b_{il}^{\pm} \\ c_{il}^{\pm} \\ d_{il}^{\pm} \end{pmatrix} \qquad \text{and} \qquad |u_s^{\pm}\rangle = \begin{pmatrix} a_s^{\pm} \\ b_s^{\pm} \\ c_s^{\pm} \\ d_s^{\pm} \end{pmatrix}. \tag{16}$$

Here a_{il}^{+}, b_{il}^{+}, c_{il}^{+}, and d_{il}^{+} (a_s^{+}, b_s^{+}, c_s^{+}, and d_s^{+}) are the amplitudes referred to the top of the lth (substrate) layer, and a_{il}^{-}, b_{il}^{-}, c_{il}^{-}, and d_{il}^{-} (a_s^{-}, b_s^{-}, c_s^{-}, and d_s^{-}) the amplitudes referred to the bottom of the lth (substrate) layer, respectively. The present treatment is an extension of that discussed in [10, ch 7.1]; this approach is formally different from those used in many theoretical studies [11, 12]. Equation (14) elucidates the fact that the amplitudes are mutually related by

$$|u_{i,l}^{-}\rangle = P_i|u_{i,l}^{+}\rangle \qquad \text{and} \qquad |u_s^{-}\rangle = P_s|u_s^{+}\rangle \tag{17}$$

with

$$P_i = P_i(-d_i) \qquad (i = 1,\ 2 \text{ or } s). \tag{18}$$

The displacement vector $(u_x^{(i)}, 0, u_z^{(i)})$ and the stress components must be continuous at the boundaries $z = -(l-1)D - d_1$ and $z = -lD$. These continuity conditions can be summarized as

$$T_1|u_{1,l}^{-}\rangle = T_2|u_{2,l}^{+}\rangle \qquad \text{and} \qquad T_2|u_{2,l}^{-}\rangle = T_1|u_{1,l+1}^{+}\rangle \tag{19}$$

using the matrix T_i ($i = 1$ or 2) given by

$$T_i = \begin{pmatrix} 1 & 1 & 1 & 1 \\ U_{i1} & -U_{i1} & U_{i2} & -U_{i2} \\ \alpha_{i1} & \alpha_{i1} & \alpha_{i2} & \alpha_{i2} \\ \beta_{i1} & -\beta_{i1} & \beta_{i2} & -\beta_{i2} \end{pmatrix} \tag{20}$$

with

$$\alpha_{ij} = \bar{C}_{13}^{(i)} + \bar{C}_{33}^{(i)} Q_{ij} U_{ij} \qquad \text{and} \qquad \beta_{ij} = \bar{C}_{44}^{(i)}(Q_{ij} + U_{ij}). \tag{21}$$

Let us write the thickness of the superlattice as $LN \cdot D$ using the total period number LN. The boundary condition between the superlattice and the substrate at $z = -z_L \equiv -LN \cdot D$ is apparently expressed as

$$T_2|u_{2,LN}^{-}\rangle = T_s|u_s^{+}\rangle \tag{22}$$

where T_s is the matrix defined by replacing i with s in equation (20). We have two surfaces at $z = 0$ and $-LN \cdot D - d_s$. From equations (17), (19), and (22), the amplitudes of the elastic waves on the top surface, $|u_{1,1}^{+}\rangle$, at $z = 0$ and those on the bottom surface, $|u_s^{-}\rangle$, at $z = -LN \cdot D - d_s$ can be related through

$$|u_s^{-}\rangle = T_s^{-1}(T_s P_s T_s^{-1})[(T_2 P_2 T_2^{-1})(T_1 P_1 T_1^{-1})]^{LN} T_1 |u_{1,1}^{+}\rangle. \tag{23}$$

The stress-free boundary conditions at the top and bottom surfaces are summarized into a compact form of

$$\begin{pmatrix} A_1 \\ A_s T_s^{-1}(T_s P_s T_s^{-1})[(T_2 P_2 T_2^{-1})(T_1 P_1 T_1^{-1})]^{LN} T_1 \end{pmatrix} |u_{1,1}^{+}\rangle = 0 \tag{24}$$

with the matrix A_i ($i = 1$ or s) defined by

$$A_i = \begin{pmatrix} \alpha_{i1} & \alpha_{i1} & \alpha_{i2} & \alpha_{i2} \\ \beta_{i1} & -\beta_{i1} & \beta_{i2} & -\beta_{i2} \end{pmatrix}. \tag{25}$$

Here α and β are given by equation (21). Equation (24) has a non-trivial solution, when the determinant of the 4×4 matrix becomes zero:

$$\det \begin{pmatrix} A_1 \\ A_s T_s^{-1}(T_s P_s T_s^{-1})[(T_2 P_2 T_2^{-1})(T_1 P_1 T_1^{-1})]^{LN} T_1 \end{pmatrix} = 0. \tag{26}$$

The elastic waves satisfying the above solubility condition (dispersion equation) are the surface waves of interest. This dispersion equation is the ultimate expression to evaluate the surface acoustic waves in various types of superlattices: a plate and a substrated superlattice of arbitrary thickness. Many theoretical works have only treated semi-infinite superlattices. However, we have, for instance, the dispersion equation for a superlattice plate by taking the $d_s \to 0$ limit, while the $d_s \to \infty$ limit gives the dispersion equation for a superlattice prepared on a substrate.

5. Results and discussions

In the previous section we have derived the dispersion equation, equation (26), for the surface waves of a superlattice prepared on a substrate with arbitrary thickness. Solving these dispersion equations, we can determine the ξ_1 values of medium 1 defined by equation (8) for a given set of ω and q_x. Defining the bulk transverse wave velocity in medium 1 by

$$v_{T,1} \equiv \left[\bar{C}_{44}^{(1)}/\rho^{(1)}\right]^{1/2} \tag{27}$$

we obtain

$$\xi_1 = \omega/(v_{T,1}q_x) = v_s/v_{T,1}. \tag{28}$$

Here v_S $(=\omega/q_x)$ is the surface wave velocity of the superlattice.

We parametrize the wavelength of the surface wave λ_S (≅3000 Å in the standard Brillouin scattering experiment) as $LD \cdot D$:

$$\lambda_S = 2\pi/q_x = LD \cdot D. \tag{29}$$

Note that the thickness of the superlattice is $LN \cdot D$ and that of the substrate is d_s.

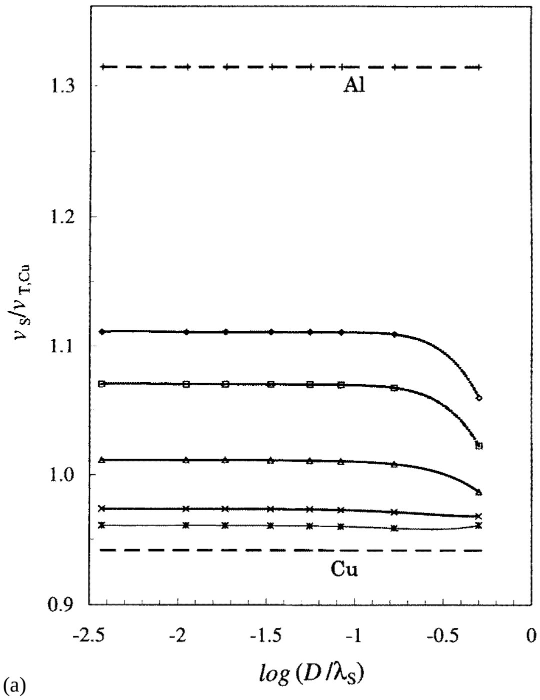
Table 2. The elastic constants $\bar{C}_{pq}^{(i)}$ [17], density $\rho^{(i)}$ [18], and transverse wave velocity $v_{T,i}$ used in the numerical calculations.

i	$\bar{C}_{11}^{(i)}$ (10^9 Pa)	$\bar{C}_{33}^{(i)}$ (10^9 Pa)	$\bar{C}_{13}^{(i)}$ (10^9 Pa)	$\bar{C}_{44}^{(i)}$ (10^9 Pa)	$\rho^{(i)}$ (10^3 kg m^{-3})	$v_{T,i}$ (10^3 m s^{-1})
Cu	22.09	23.82	8.74	4.08	8.96	2.13
Al	11.40	11.57	5.97	2.47	2.70	3.02
Ag	15.16	16.15	7.23	2.49	10.50	1.54
Glass [19]	7.85	7.85	1.61	3.12	2.20	3.77

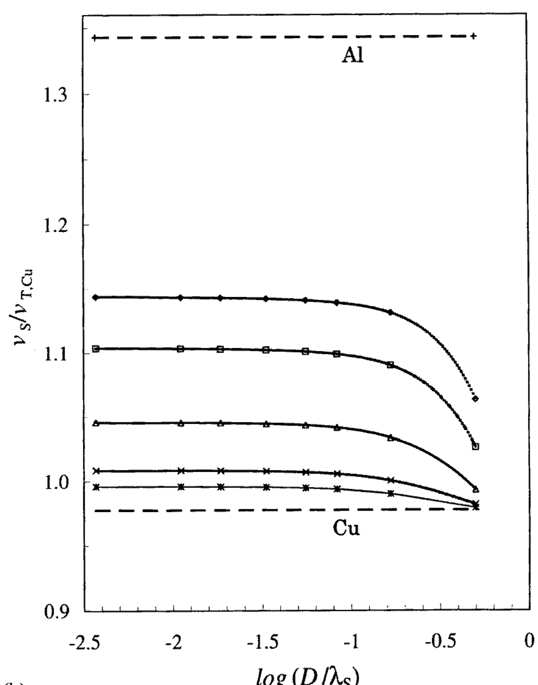
Numerical calculations on the Rayleigh wave velocity using equation (26) for an arbitrary substrate thickness d_s and infinite thickness ($d_s \to \infty$) give almost the same velocities for $d_s/LD \cdot D \geqslant 3.5$: a substrate thicker than three times the wavelength of the surface wave can be regarded as an infinite thickness substrate (∼1 μm in the standard Brillouin scattering experiment).

Here we calculate the surface wave velocities of sputtered Cu/Al and Cu/Ag superlattices, which consist of randomly oriented polycrystalline layers stacked upon the (111) planes. The elastic constants $\bar{C}_{pq}$, which are isotropic in the (x, y) plane, are given in table 2. We will assign the thickness of a Cu layer to d_1 and that of an Al layer or an Ag layer to d_2. We have performed numerical calculations both for superlattice plates and superlattices prepared on glass substrate.

The surface wave velocity of a superlattice plate can be evaluated from equation (26) taking the limit of $d_s \to 0$, while that for a superlattice on glass substrate of infinite thickness can be evaluated by taking the $d_s \to \infty$ limit. In addition, using the dispersion equation (26),



(a)



(b)

Figure 2. (a) Relative surface wave velocities $v_S/v_{T,Cu}$ in Cu/Al superlattice plates of $1.5\lambda_S$ thickness. The broken lines are the velocities of pure Cu and pure Al. From top to bottom, the relative velocities in the superlattice plates with thickness ratio $d_1\colon d_2 = 1\colon 3$, 1:2, 1:1, 2:1, and 3:1, respectively. (b) Relative surface wave velocities $v_S/v_{T,Cu}$ in Cu/Al superlattices of $1.5\lambda_S$ thickness on glass substrate. The symbols are as in (a).

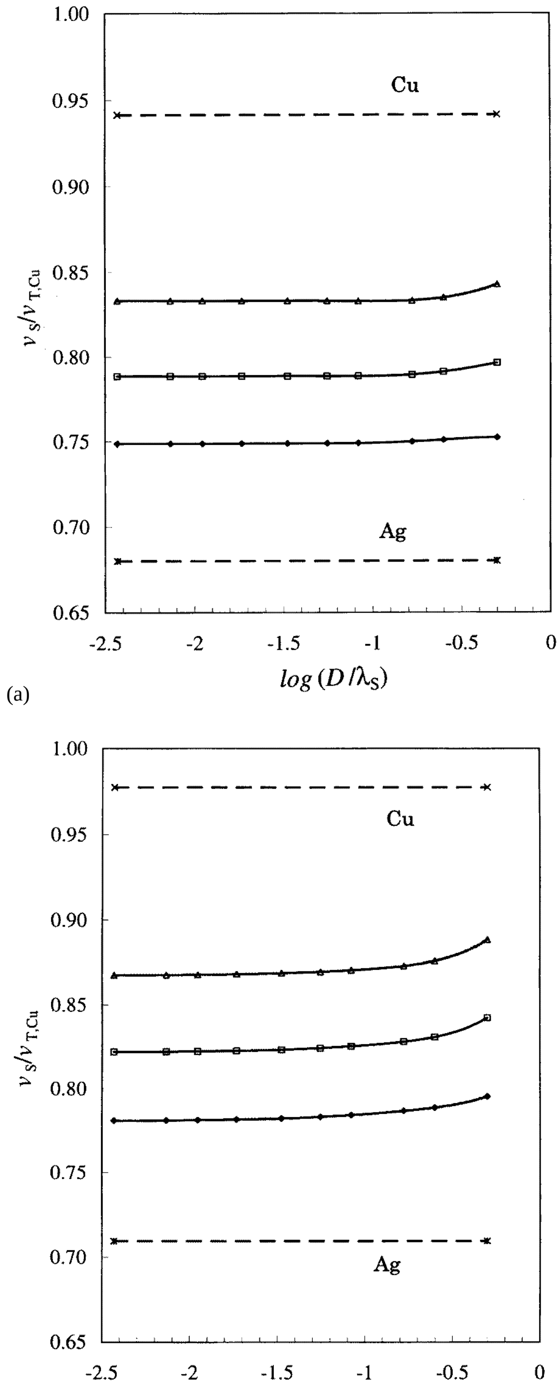


Figure 3. (a) Relative surface wave velocities $v_S/v_{T,Cu}$ in Cu/Ag superlattice plates of $1.5\lambda_S$ thickness. The broken lines are the velocities of pure Cu and pure Ag. From top to bottom, the relative velocities in the superlattice plates with thickness ratio $d_1 : d_2 = 2 : 1$, 1:1, and 1:2, respectively. (b) Relative surface wave velocities $v_S/v_{T,Cu}$ in Cu/Ag superlattices of $1.5\lambda_S$ thickness on glass substrate. The symbols as in figure 2(a).

Table 3. (a) Relative surface wave velocities in Cu/Al superlattice plates ($LN/LD = 1.5$). Pure Cu: 0.94169, pure Al: 1.31428, ∞: EEC model, Cu:Al $= d_1 : d_2$. (b) Relative surface wave velocities in Cu/Al superlattices on glass ($LN/LD = 1.5$). Pure Cu: 0.97754, pure Al: 1.34300, ∞: EEC model, Cu:Al $= d_1 : d_2$.

(a)

LD	Cu:Al = 1:3	Cu:Al = 1:2	Cu:Al = 1:1	Cu:Al = 2:1	Cu:Al = 3:1
2	1.05963	1.02250	0.98685	0.96801	0.96099
6	1.10921	1.06751	1.00825	0.97120	0.95874
12	1.11084	1.06982	1.01056	0.97285	0.96010
18	1.11094	1.07011	1.01102	0.97330	0.96050
30	1.11097	1.07023	1.01127	0.97356	0.96073
54	1.11097	1.07028	1.01137	0.97366	0.96083
90	1.11097	1.07029	1.01139	0.97369	0.96086
270	1.11098	1.07030	1.01140	0.97371	0.96087
∞	1.11098	1.07030	1.01141	0.97371	0.96087

(b)

LD	Cu:Al = 1:3	Cu:Al = 1:2	Cu:Al = 1:1	Cu:Al = 2:1	Cu:Al = 3:1
2	1.06316	1.02616	0.99305	0.98184	0.97915
6	1.13119	1.09002	1.03343	1.00028	0.98989
12	1.13910	1.09888	1.04144	1.00558	0.99371
18	1.14094	1.10088	1.04327	1.00688	0.99469
30	1.14223	1.10223	1.04445	1.00771	0.99532
54	1.14304	1.10303	1.04511	1.00816	0.99567
90	1.14344	1.10341	1.04541	1.00836	0.99582
270	1.14383	1.10378	1.04570	1.00855	0.99595
∞	1.14403	1.10396	1.04583	1.00863	0.99601

we can treat a superlattice of arbitrary thickness through adjustment of the ratio LN/LD, where LD indicates the number of periods included in the surface wavelength λ_S defined by equation (29), while LN corresponds to the sum of the periods of the superlattice. The results for Cu/Al superlattices with $LN/LD = 1.5$, which corresponds to a thickness of ~4500 Å, are summarized in figures 2(a) and (b) and tables 3(a) and (b). Here the surface wave velocities relative to the bulk transverse wave velocity of Cu, $v_S/v_{T,Cu}$, are shown. The surface wave velocities for the Cu/Ag superlattices are shown in figures 3(a) and (b) and tables 4(a) and (b). It is obvious from tables 3(a) and 4(a) and figures 2(a) and 3(a) that we can expect a constant velocity for short-period (large LD) superlattices. This indicates that a short-period superlattice has its own elastic constants as Grimsditch has previously pointed out [8].

Now we regard a superlattice plate and a superlattice on glass substrate as an effective medium of thickness $LN \cdot D$ with the EECs $\bar{C}_{11}^{(e)}$, $\bar{C}_{33}^{(e)}$, $\bar{C}_{13}^{(e)}$, and $\bar{C}_{44}^{(e)}$. The amplitudes of the displacements for elastic waves $|u_e^+\rangle$ and $|u_e^-\rangle$ satisfy

$$|u_e^-\rangle = P_e|u_e^+\rangle. \tag{30}$$

Here P_e is the matrix defined by $P_e = P_e(-LN \cdot D)$, from equation (18) replacing i by e with definition (15). The variable Q_{ej} ($j = 1$ or 2) can be calculated from equation (10), by substituting the EECs into equation (11). The dispersion equation is given by

$$\det\begin{pmatrix} A_e \\ A_s T_s^{-1}(T_s P_s T_s^{-1})(T_e P_e T_e^{-1})T_e \end{pmatrix} = 0. \tag{31}$$

Table 4. (a) Relative surface wave velocities in Cu/Ag superlattice plates ($LN/LD = 1.5$). Pure Cu: 0.94169, pure Ag: 0.67996, ∞: EEC model, Cu:Ag $= d_1 : d_2$. (b) Relative surface wave velocities in Cu/Ag superlattices on glass ($LN/LD = 1.5$). Pure Cu: 0.97753, pure Ag: 0.70942, ∞: EEC model, Cu:Ag $= d_1 : d_2$.

(a)

LD	Cu:Ag = 1:2	Cu:Ag = 1:1	Cu:Ag = 2:1
2	0.75213	0.79633	0.84282
4	0.75080	0.79115	0.83509
6	0.74971	0.78932	0.83334
12	0.74890	0.78859	0.83306
18	0.74876	0.78856	0.83316
30	0.74869	0.78857	0.83323
54	0.74866	0.78858	0.83327
90	0.74866	0.78858	0.83328
136	0.74865	0.78858	0.83328
270	0.74865	0.78858	0.83329
∞	0.74865	0.78858	0.83329

(b)

LD	Cu:Ag = 1:2	Cu:Ag = 1:1	Cu:Ag = 2:1
2	0.79533	0.84235	0.88837
4	0.78862	0.83084	0.87597
6	0.78676	0.82818	0.87309
12	0.78421	0.82543	0.87059
18	0.78315	0.82434	0.86968
30	0.78224	0.82340	0.86889
54	0.78163	0.82275	0.86833
90	0.78132	0.82242	0.86804
136	0.78116	0.82225	0.86790
270	0.78101	0.82208	0.86775
∞	0.78086	0.82192	0.86761

Here A_e and T_e can be readily obtained by replacing the variables in equations (25) and (20). Thus we can calculate the surface wave velocities for a plate and a substrated film with the EEC model. The results are listed in the columns $LD = \infty$ in tables 3(a), (b) and 4(a), (b). As mentioned already, LD is the number of the period D contained in the wavelength of the surface wave. The EEC model corresponds to the case of $LD = \infty$, because the model is based on the assumption that the period is infinitesimal. We can confirm from these tables that the EEC model surely gives the surface wave velocities in the limit of the zero period ($D \to 0$ or $LD \to \infty$).

Tables 3(a) and 4(a) show how the surface wave velocity of the superlattice plate approaches the value calculated from the EEC model as the period decreases. It is apparent from these tables that the superlattice plates with LD values larger than 50 can be regarded as a plate with the EECs with excellent accuracy. The EEC model is obviously reasonable for $LD \geqslant 12$, within an error less than 0.1%. We can expect an increasing error when the EEC model is applied to superlattice plates with an LD value of less than 10.

Tables 3(b) and 4(b) indicate how the surface wave velocity in the substrated superlattices approaches the value from the EEC model. The surface wave velocity is rather susceptible to the LD parameter, in contrast with the case of the corresponding plates. The convergence to the EEC velocity is much slower and the velocity obtained from the shortest period film ($LD = 270$) is still approaching the EEC value. However, the EEC model can be applicable for substrated superlattices with $LD \geqslant 12$, if one can ignore a possible error of up to several per cent.

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